Self-propelled hard disks: non-conservation of momentum and transition to collective motion

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A self-propelled particle borrows energy from its surrounding environment and converts it to translational motion. A system composed of such self-propelled particles is inherently far from equilibrium, in a very peculiar way: the property of detailed balance is broken at the level of every single particle. As of the collective level, such systems are able to take energy from the microscopic scale and inject it into the macroscopic motion of the whole system, thus sustaining an orientationally ordered stationnary state in two dimensions (see Fig. 1b). Such a polar state cannot exist for a system in equilibrium. If we start from this polar state and increase the orientational noise, the stationnary state becomes isotropic. Inbetween, a polar–isotropic phase transition generically occurs at intermediate orientational noise.

The current understanding of the polar–isotropic phase transition mostly relies on kinetic theories, at least for the homogeneous case, on which we restrict ourselves. Making use of the molecular chaos assumption, these theories are able to describe quantitatively the diluted regime; hydrodynamic equations can be derived [2]. It is well understood that the phase transition is controlled qualitatively by the competition between the so-called "alignment" of interacting self-propelled particles, which tends to destabilize the isotropic phase, and the orientational noise, which tends to stabilize it.

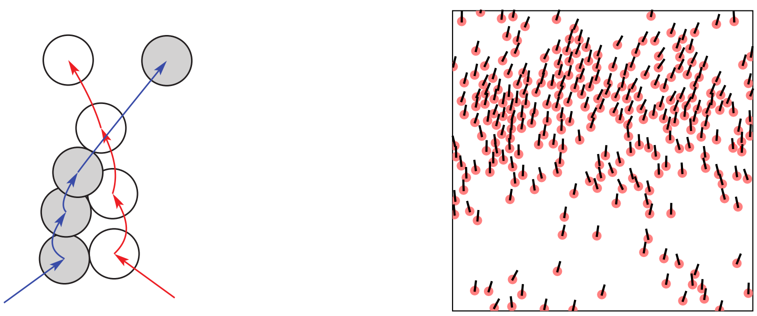

Figure 1: (a): The scattering of two self-propelled hard discs can be made of several hard-core collisions. (b): Collective directed motion of many self-propelled hard discs.

In a recent work [1], we were able to formulate a kinetic theory in terms of microscopic quantities describing the non-conservation of momentum arising from binary interactions. To be more specific, let us consider two particles that come in interaction, having a pre-scattering net momentum $\mathbf{p}$. After the interaction, when particles are far enough one from the other, the scattering event is over, both particles go straight and their summed momentum has become $\mathbf{p}'$. We show that the "alignment" resulting from the interaction can be properly defined as the microscopic quantity $\mathbf{p} \cdot (\mathbf{p}' - \mathbf{p})$, which has a nice geometric interpretation. Of course, this quantity depends on the pre-scattering parameters of the interaction (such as the incoming relative velocity and the impact parameter). At the collective level, one needs to consider averages of $\mathbf{p} \cdot (\mathbf{p}' - \mathbf{p})$ over the space of pre-scattering parameters, weighted by the appropriate scattering rate derived from statistical mechanics arguments. Such averages give access to quantities of interest, providing predictions for the transition, whether it is a continuous or discontinuous one, the scattering rate in the polar state, the fluctuations of the order parameter in the isotropic state.

We will present these results in more details and illustrate them by comparing their predictions to numerical simulations of a model of self-propelled hard discs that was shown to describe very well the data of a recent experiment [3].

References

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