Quantum hydrodynamics and its applications to dense astrophysical plasmas

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Astrophysical objects such as neutron stars, pulsars, magnetars, and white dwarfs constitute a unique example of matter under extreme conditions of temperature and density. In such exotic environments, matter can only exist in the form of plasma for which, however, quantum effects play a significant role because of the very high densities involved.

A lot of progress has been made in classical plasma physics using both kinetic and fluid models. These models generally neglect quantum effects, which are negligible for most applications (with the possible exception of plasmas generated through laser-solid interactions). On the other hand, condensed matter theorists have developed a battery of tools to tackle the quantum dynamics of charged particles (electrons) in metals and semiconductors. In these contexts, time-dependent Hartree-Fock methods and density functional theory (DFT) can furnish a very accurate description of the electron response in solids, but require substantial computational resources to simulate realistic systems. The main reason for this computational complexity is that these methods are based on wave functions.

Alternatively, it is possible to develop a quantum hydrodynamic (QHD) approach which does not rely on the wave functions, but rather on a small number of macroscopic quantities, such as the particle density, momentum, and energy [1-3]. The QHD model is made of a set of conservation equations for the density (continuity equation), the momentum, and the energy or pressure, coupled to Poisson's equation for the electric potential. They can be derived, with suitable approximations, from the equations of time-dependent DFT, and contain quantum corrections to the lowest order. From the computational point of view, the hydrodynamic approach leads to considerable gains in computing time in comparison with simulations based on conventional methods such as DFT.

In this paper, we will present the basic hypotheses underlying the QHD approach and its current use in condensed matter physics [4]. We will also discuss its possible applications to extreme astrophysical environments [5].

Finally, we will illustrate two extensions of the QHD theory that are also relevant to astrophysics: the inclusion of spin and relativistic effects.

Relativistic effects are ubiquitous in high-density astrophysical environments, because of the very large energies that are involved. A semi-relativistic version of the QHD model can be derived from the Dirac equations by taking an appropriate expansion in powers of 1/c [6-8]. On the other hand, spin effects [9, 10] are naturally linked to relativistic quantum mechanics, and appear even at the lowest order in these expansions. At higher orders, spin and orbital motion are intrinsically coupled.

Some results involving spin and relativistic effects will be shown and their relevance to astrophysical plasmas will also be discussed.

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