Wakes and wave-resistance on viscous thin-films

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A disturbance moving above a liquid or soft interface deforms its shape, thereby producing a wake. This is a common phenomenon, which may take place in systems with sizes of different order of magnitude, going from atomic force microscopy configurations, (the intrusive effect of local probes on soft samples [1]) and mesoscopic scales (the consequences of local steady disturbances occurring during coating procedures [2]), to macroscopic phenomena (molten and fluidized materials involved in natural disasters [3,4]).

The effect of a pressure disturbance, being displaced with a constant speed V, at the surface of a thin-film, has been studied analytically for two and three-dimensional geometries. The disturbance always creates a depression on the surface of the film that relaxes towards a flat surface in the far-field. Nevertheless, the way this relaxation occurs depends strongly on the disturbance speed. Nearly symmetric surface shapes are found in the low speed regime, whereas asymmetric surface profiles are discerned for high disturbance speeds. For the 2D case, a surface oscillation is observed in the front, whereas an exponential relaxation occurs in the rear. For the 3D case, a hump is observed enclosing the front of the disturbance projection over the film, which, as it turns around, becomes a wake in the rear. A physical reasoning, supported by a 2D asymptotic solution, has been done to explain the existence of symmetric and non-symmetric profiles in both geometries.

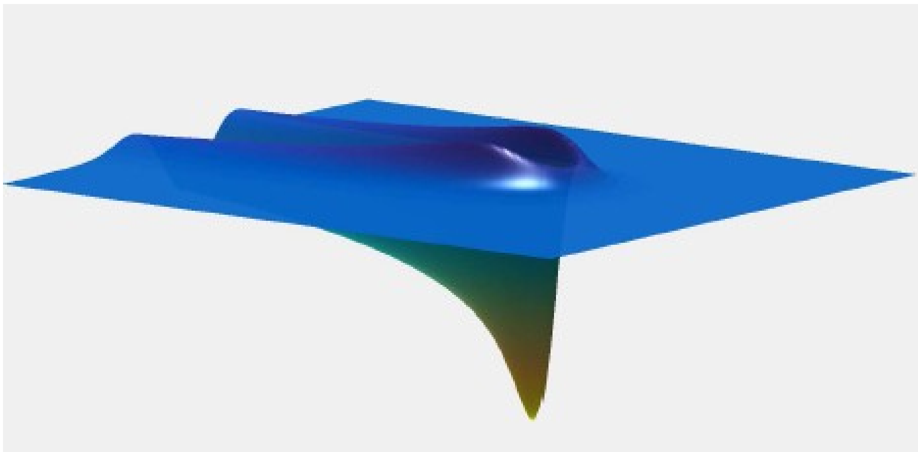

Figure : Two-dimensional surface profile of a thin-film subjected to pressure disturbance traveling at a relatively high-speed.

The wake angle and the wave pattern, generated behind a disturbance sailing at the surface of an inviscid liquid, have been addressed more than a hundred years ago and remain a topic of interest [5,6]. The generated wake continually consumes energy, which is transported away from the disturbance. This energy is traduced into a force, called wave-resistance [7], which is experienced by the disturbance and needs to be furnished to ensure a constant velocity. The wave-resistance caused by the capillary-gravity waves, has been fully characterized for an incompressible inviscid liquid [8].

In the presented work, the wave-resistance R has been calculated in the viscous thin-film context. For finite size pressure disturbances, the generated wave-resistance tends to increase with the speed as $R \sim V$, up to a certain transition value above which a monotonic decrease of this force occurs, as $R \sim 1/V$. Several simple expression for the wave-resistance have been retrieved for the different speed regimes, including the influence of the other relevant parameters: the Bond number $B_o = \sqrt{\rho g a^2 / \gamma}$, (where ρ and γ are the density difference and the surface tension of the air/liquid interface, g the acceleration of gravity and a the size of the pressure distribution) and $P_0 = p_0 / \rho g H$ the dimensionless intensity of the pressure distribution (where p_0 is the intensity with dimensions and H the thickness of the liquid film). The role of size has been analyzed in depth, revealing that the transition speed scales as $V \sim B_o^{-\alpha}$, where $\alpha > 0$. Therefore, for small disturbances the regime transition occurs at higher speeds and vice-versa. Coherently, for a point-like disturbance, the wave-resistance either saturates to a constant value (for a 2D geometry) or diverges as $R \sim V^{1/3}$ (for a 3D geometry).

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